A Hybrid Algorithm for the Synthesis of Distributed Antenna Arrays with Excitation Range Control

Magdy A. Abdelhay

Al-Farqadein University, Basrah, Iraq

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Abstract — Excitation coefficients with a low dynamic range ratio (DRR) are advantageous in controlling mutual coupling between the elements of an antenna array. Their use also reduces the output power loss and simplifies the design of the feeding network. In this paper, a hybrid algorithm based on invasive weed optimization and convex optimization for the synthesis of distributed arrays with two subarrays is proposed. Arrays of this type are used in numerous applications, e.g. in aircraft. A constraint is added to the optimization problem to control the DRR of the array's excitation vector. Numerical results are presented for position-only, as well as for position and excitation control approaches. The trade-off between the peak sidelobe ratio and the obtained DRR is illustrated by numerical examples.

Keywords — convex optimization, distributed antenna arrays, dynamic range ratio, invasive weed optimization

1. Introduction

A distributed phased array (DPA) is composed of multiple small-scale arrays, which increases the array's arrangement flexibility and expands the its aperture. DPA with a large aperture offers highly favorable characteristics, such as high directivity and narrow mainlobe width. Due to these features, DPA finds use in many applications in communication systems relying on special layout platforms [1], [2], and in other applications which cater to the high demand for good directivity and great precision with increased degrees of freedom [3], [4].

A DPA is ordinarily a sparse array with nodes that can be placed on independent platforms tens of wavelengths apart. This leads to the appearance of grating lobes in the array's pattern. It is essential in many applications to suppress these grating lobes to avoid problems such as interference from undesired locations.

Many synthesis techniques have been proposed to suppress grating lobes in DPAs [5]–[7]. The synthesis process depends on numerous parameters, including position, excitation, and the number of array elements. Many array pattern synthesis techniques employ global optimization techniques, such as genetic [8], invasive weed optimization (IWO) [9], differential evolution [10], and particle swarm optimization [11] algorithms. Convex optimization has also been widely used to synthesize antenna arrays [12]. Compressive sensing-based approaches have been utilized in [13]–[15] for this purpose as well.

Dynamic range ratio (DRR) is defined as the ratio of the array elements' amplitudes at maximum and minimum values. The DRR of the excitation coefficients is usually high in the synthesized arrays with a low sidelobe level (SLL) [16], [17]. High DRR is undesirable, since it complicates the feeding network and increases its cost. Furthermore, low DRR results in better control of the mutual coupling between antenna elements. Many analytical methods based on popular windows and polynomials, for example Gaussian [18] and ultraspherical windows [19], are used to synthesize array patterns with low DRR. Optimization-based methods, which include the need for low DRR as a design objective, are also used to synthesize arrays with low DRR of the excitations [20], [21].

In [22], a hybrid algorithm for synthesizing a distributed array consisting of two subarrays using differential evolution and convex optimization was proposed. In this proposed method, the differential evolution algorithm is used to find the element positions and the iterative reweighted ℓ_1-norm minimization algorithm is employed to find the optimum weights for a given set of element positions. Unfortunately, the use of iterative reweighted ℓ_1-norm minimization is not necessary, as it is usually relied upon to enhance the sparsity in solutions for optimization problems which use ℓ_1-norm instead of ℓ_0 quasi-norm to minimize the number of non-zero elements in the excitation vector [23]. It is not used to further lower the peak sidelobe level (PSLL), as mentioned in [22].

In the case of the work described in [22], the optimization problem for a given position vector which is obtained using the differential evolution algorithm is convex, and there is no need for any relaxation. Furthermore, the results reported in the paper, i.e. those shown in Tab. 1 in [22], did not satisfy the constraint on the distance between the two subarrays, which should be 30λ instead of the reported 18λ. The work also did not consider the DRR of the excitations of the synthesized array.

In this paper, an algorithm based on IWO and convex optimization is proposed to synthesize distributed arrays consisting of two subarrays, with DRR taken into consideration as well. In the proposed algorithm, IWO is used to find the optimum positions of the array's elements under a constraint on the distance between the two subarrays and a minimum allowed distance between 2 adjacent array elements.

Convex optimization is used to find the optimum excitation vector for a given set of element positions, which minimizes PSLL, with a constraint aimed at minimizing DRR of the excitations. PSLL of the synthesized array is used as the fitness function for the IWO algorithm. To the best of the author's knowledge, this is the first paper focusing on the synthesis of distributed antenna arrays with constraints on the distance between the sub-arrays and the inter-element spacing between the elements in each sub-array, with dynamic range ratio considerations accounted for as well.

The remainder of the paper is organized as follows. Section 2 formulates the problem. The proposed algorithm is detailed in Section 3. Numerical examples are given in Section 4, and conclusions are drawn in Section 5.

2. Synthesis Problem Formulation

Consider a linear array made up of two identical sub-arrays which consist of $2 \times M$ isotropic radiating elements, with the distance between the sub-arrays equaling D_0. The distance between the individual elements in the same subarray is d_0. The location of the n-th array element x_n can be expressed as:

$$x_n = \begin{cases} -(N-n)\, d_0 - \frac{D_0}{2}, & 1 \leqslant m \leqslant M \\ \frac{D_0}{2} + (n-N-1)\, d_0, & N+1 \leqslant n \leqslant 2N \end{cases} . \tag{1}$$

The distance between two elements on the left-hand side of each subarray equals:

$$x_{N+1} - x_1 = D_0 + (N-1)\, d_0 . \tag{2}$$

The array's far field pattern can be written as:

$$AF(\theta) = \sum_{n=1}^{2N} w_n\, e^{-jkx_n \sin\theta} , \tag{3}$$

where w_n is the excitation of the n-th element, $k = \frac{2\pi}{\lambda}$ is the wave number, λ is the wavelength, and θ is the elevation angle. Equation (3) can be written in a matrix form as:

$$AF(\theta) = \boldsymbol{A}(\theta)^T \boldsymbol{w} , \tag{4}$$

where T is the transpose operator,

$$\boldsymbol{A}(\theta) = \left[e^{-jkx_1 \sin\theta},\, e^{-jkx_2 \sin\theta}, \ldots,\, e^{-jkx_{2N} \sin\theta}\right]^T$$

and

$$\boldsymbol{w} = [w_1,\, w_2,\, \ldots,\, w_{2N}]^T .$$

The objective here is to find element locations and excitations that minimize the peak sidelobe level (PSLL), subject to constraints on the number of elements, minimum element separation, and a fixed distance between the two sub-arrays. Mathematically, the optimization problem can be expressed as:

$$\begin{cases} \text{find } \boldsymbol{x} = [x_1, \ldots, x_{2N}]^T \text{ and } \boldsymbol{w} = [w_1, \ldots, w_{2N}]^T \\ \min \{\text{PSLL}(\boldsymbol{x}, \boldsymbol{w}), \text{DRR}(\boldsymbol{w})\} \\ \text{subject to } \; x_{i+1} - x_i \geqslant d_c > 0 \\ \qquad i \in \mathbb{Z},\; 1 \leqslant i \leqslant 2N-1,\; i \neq N \\ \qquad x_{N+1} - x_N \geqslant D_0 > 0 \\ \qquad x_0 = 0 \end{cases} , \tag{5}$$

where d_c is the minimum allowable distance between elements in each sub-array and the PSLL is defined as:

$$\text{PSLL}(\boldsymbol{x}, \boldsymbol{w}) = \max \left| \frac{\sum_{n=1}^{2N} w_n\, e^{-jkx_n \sin(\theta_{\text{sl}})}}{AF(\theta_0)} \right| , \tag{6}$$

where θ_0 is the direction of the mainlobe, θ_{sl} is the sidelobe angles outside of the mainlobe region, and $|\cdot|$ is the absolute value.

The DRR is defined as:

$$\text{DRR} = \frac{\max\{|\, w_k \,|\}}{\min\{|\, w_k \,|\}}, \qquad k = 1, 2, \ldots, 2N , \tag{7}$$

which represents the ratio of maximum and minimum values for the amplitudes of the array's elements.

3. The Proposed Hybrid Method

Hybrid IWO and convex optimization algorithms are used to solve the optimization problem in Eq. (5). The proposed algorithm is summarized below.

3.1. Element Position Initialization

The individual here is taken as the position vector $\boldsymbol{x} = [x_1, \ldots, x_{2N}]^T$. For the sake of satisfying the constraints on the minimum spacing between the elements in each subarray d_0 and the space between the two subarrays D_0, the position vector is expressed as follows:

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_N \\ x_{N+1} \\ x_{N+2} \\ \vdots \\ x_{2N} \end{bmatrix} = \begin{bmatrix} 0 \\ a_1 \\ a_2 \\ \vdots \\ a_{N-1} \\ a_N \\ a_{N+1} \\ \vdots \\ a_{2N-1} \end{bmatrix} + \begin{bmatrix} 0 \\ d_0 \\ 2\, d_0 \\ \vdots \\ (N-1)\, d_0 \\ (N-1)\, d_0 + D_0 \\ Nd_0 + D_0 \\ \vdots \\ (2N-2)\, d_0 + D_0 \end{bmatrix} \tag{8}$$

The vector $\boldsymbol{a} = [a_1, \ldots, a_{2N-1}]^T$ consists of $2N-1$ real random numbers in the range of $[0, \text{V}_{\max}]$, and elements of $\boldsymbol{a}$ are ordered in ascending order, i.e. $a_1 \leqslant a_2 \leqslant \ldots \leqslant a_{2N-1}$.

The position vector $\boldsymbol{x}$ can be determined after generating $\boldsymbol{a}$. Here $\boldsymbol{a}$ is considered the seed for the IWO algorithm. By producing $\boldsymbol{a}$ M times independently, a starting population of M seeds is initialized. Consequently, a set of M position vectors are initialized.

3.2. Fitness Function

Provided that the positions of the array elements are determined by the IWO algorithm, the optimization problem in Eq. (5) is a convex optimization problem which can be solved efficiently using off-the-shelf packages, such as CVX [24]. In such a case, the optimization problem can be expressed mathematically as:

$$\min_{w,\tau_s} \tau_s \tag{9a}$$

$$\text{subject to } \operatorname{Re}\left\{\boldsymbol{A}(\theta_0)^T \boldsymbol{w}\right\} = \tau_m \tag{9b}$$

$$\left|\boldsymbol{A}(\theta_{\text{sl}})^T \boldsymbol{w}\right| \leqslant \tau_s \tag{9c}$$

$$\|\boldsymbol{w}\| \leqslant \tau_d \tag{9d}$$

where $\operatorname{Re}\{\cdot\}$ is the real part.

Without normalization, τ_m is the directivity of original distributed array and τ_s is a slack variable which represents an upper bound on the response of the array in the sidelobe region. $\|\cdot\|$ is the ℓ_2-norm, which is the square root of the sum of the squared values of the vector elements. τ_d represents an upper on the ℓ_2-norm of the excitation vector $\boldsymbol{w}$.

Unlike the ℓ_1-norm, the ℓ_2-norm does not promote sparsity in solutions. Instead, it distributes the penalty across all coefficients, resulting in more evenly distributed values. This leads to a reduction in the ratio between the largest and smallest values that element excitations can assume, which results in a decrease in the DRR.

The resulting PSLL of the array is considered to be the fitness value of the correspondent seed in the population. Every initial seed grows into a weed after calculating its fitness.

3.3. Reproduction

The reproductive capability of weeds depends on their fitness values. A linear relationship exists between the number of seeds reproduced from every weed and its fitness value, i.e. PSLL associated with the weed. Here, the weeds with lower fitness values have a larger probability of being preserved in the population and, hence, produce more seeds. The number of seeds produced by the m-th weed can be expressed as:

$$s_m = \frac{S_{\text{max}} - S_{\text{min}}}{f_{\text{max}} - f_{\text{min}}}(f_{\text{max}} - f_{\text{m}}) + S_{\text{min}} \,, \tag{10}$$

where f_{max} and f_{min} are the maximum and minimum fitness values, i.e. PSLLs, in the current population, respectively. S_{max} and S_{min} are the maximum and minimum allowable seeds, respectively. f_{m} is the fitness value of the m-th weed.

3.4. Spatial Dispersal

New seeds are then dispreaded in a random manner over the searching space. Gaussian distribution is used with mean μ equal to the location of the parent weed. During the iterations, the standard deviation σ is reduced from its initial maximum value σ_{initial} to its final minimum value σ_{final}. The value of σ during iteration i can be calculated using the relation:

$$\sigma = \frac{(i_{\text{max}} - i)^n}{(i_{\text{m}})^n}(\sigma_{\text{initial}} - \sigma_{\text{final}}) - \sigma_{\text{final}} \,, \tag{11}$$

where n is a nonlinear modulation index and i_{max} is the maximum number of iterations.

Then the k-th seed produced by the m-th weed may be written as:

$$\boldsymbol{a}_{n,k} = \boldsymbol{a}_n + \mathcal{N}(0,\sigma) \,. \tag{12}$$

Following that, the elements of each seed $\boldsymbol{a}$ are limited in the range of $[0, \text{V}_{\text{max}}]$ and thus ordered in an increasing order $a_1 \leqslant a_2 \leqslant \ldots \leqslant a_{2N-1}$.

Equation (8) is thus used to calculate the corresponding position vector, and next (9) is used to find the optimum excitation vector which minimizes the PSLL of the distributed array pattern.

3.5. Competitive Exclusion

The weeds are grown from the seeds and ranked together with parent weeds based on their PSLL fitness value. As the number of weeds increases, there must be some sort of competition between them to limit their maximum number in the colony. When the maximum number of weeds p_{max} is reached, weeds with poor fitness, i.e., with their PSLL being high in comparison to that of other weeds, are removed from the current colony. On the other hand, the weeds with better fitness will survive and be allowed to reproduce their next generations. The process is repeated as described in Subsection 3.3 until the termination process criteria are met, i.e., the number of maximum iterations i_{max} is reached.

Tab. 1. List of element positions for $N = 25$ element array with position-only control.

n	Pos. (λ)	n	Pos. (λ)	n	Pos. (λ)	n	Pos. (λ)
1	0	14	12.3381	27	40.7146	40	49.7632
2	0.7003	15	12.9280	28	41.2152	41	50.8395
3	1.3394	16	14.3832	29	41.9873	42	52.4788
4	1.8571	17	15.4114	30	42.7831	43	53.1647
5	3.0920	18	16.2914	31	43.3900	44	54.0020
6	3.6944	19	16.8024	32	43.9927	45	54.7090
7	4.7633	20	17.5651	33	44.5261	46	56.7148
8	5.5138	21	18.1179	34	45.0555	47	57.3373
9	6.7853	22	18.6586	35	45.5993	48	58.0154
10	7.5022	23	19.1612	36	46.1893	49	58.7423
11	8.2622	24	19.7004	37	47.1063	50	59.3086
12	9.0797	25	20.2101	38	48.4132		
13	11.4735	26	40.2112	39	49.1710		

4. Simulation Results

4.1. Position-only Control

Consider an array of 50 elements, which consists of two subarrays, each containing $N = 25$ elements. The distance

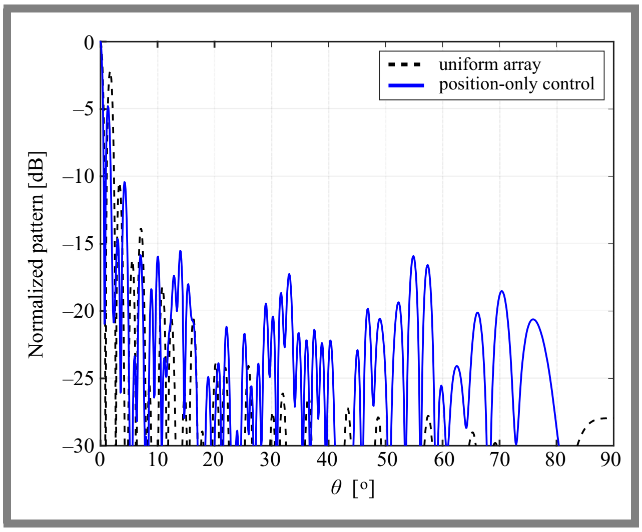

Fig. 1. Patterns of the original uniformly spaced array vs. the array with uniform amplitudes and optimized element positions.

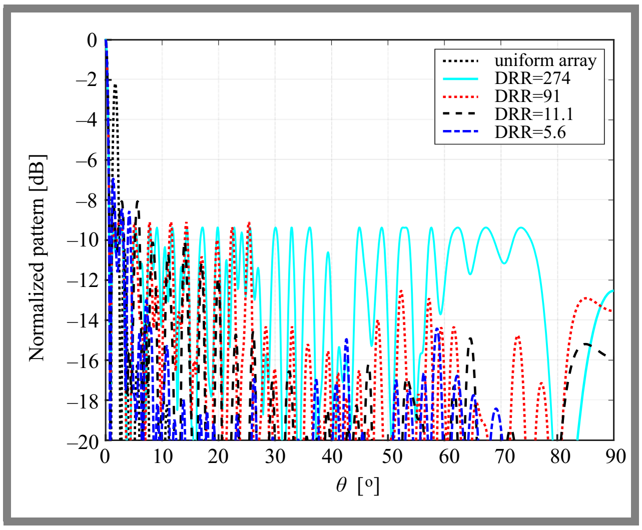

Fig. 2. Patterns of the synthesized arrays with with D_0 = 20λ.

between subarrays is $D_0 = 20\lambda$, and the distance between the elements in each subarray is $d_0 = 0.5\lambda$. The array with uniform amplitudes and fixed spacing between the elements has a PSLL of –2.17 dB for the normalized pattern. Optimizing only the positions of the array elements resulted in an array with a PSLL of –4.79 dB for the normalized pattern. The first null beam width (FNBW) of array pattern is 1.8°. A list of the position of each element is given in Tab. 1.

It can be seen from the list that the distance between each successive elements is greater than or equal to $d_0 = 0.5\lambda$ and the distance between the two subarrays equals to $D_0 = 20\lambda$. Therefore, the constraints on the optimization problem are satisfied in the synthesized array. The normalized patterns of the uniformly spaced array and the synthesized array with optimized element locations are depicted in Fig. 1.

4.2. Position and Excitation Control

The same array as described in Subsection 4.1 ($N = 25$, $d_0 = 0.5\lambda$ and $D_0 = 20\lambda$) is considered here. The array is synthesized by optimizing both the positions and excitations of the array elements. We start with optimizing the array using the objective function given in (9a) under the constraints defined Eqs. (9b) and (9c) only. That is, there is no constraint on the the ℓ_2-norm of the weight vector $\boldsymbol{w}$. The resultant array has a PSLL of –9.38 dB with a DRR of 274. Table 2 contains

Tab. 2. List of element positions and normalized excitations for $N = 25$ element array with no constraint of the weight vector $\boldsymbol{w}$.

n	Position (λ)	w_n	n	Position (λ)	w_n
1	0	0.0454	26	37.1519	1.0000
2	0.5338	0.0867	27	38.3303	0.1541
3	1.3945	0.0710	28	39.3940	0.2147
4	2.0561	0.0518	29	40.2403	0.0037
5	2.6577	0.0095	30	41.0885	0.0000
6	3.3152	0.0000	31	42.5882	0.1567
7	3.9472	0.0941	32	43.7477	0.1770
8	4.4813	0.0647	33	44.2883	0.0836
9	5.2003	0.0645	34	45.9830	0.0000
10	5.8500	0.0591	35	46.5271	0.0782
11	6.8806	0.0000	36	47.0757	0.1670
12	7.7895	0.1113	37	47.6219	0.0419
13	8.6952	0.1279	38	48.9694	0.0489
14	9.4475	0.1163	39	49.7959	0.0700
15	9.9618	0.0000	40	50.9733	0.1224
16	10.9504	0.0000	41	52.3476	0.0000
17	11.5424	0.0887	42	52.9467	0.0604
18	12.3939	0.1194	43	53.7691	0.0000
19	12.9026	0.0000	44	54.8033	0.0952
20	13.5500	0.0000	45	55.5401	0.0836
21	14.0602	0.0000	46	56.1475	0.1136
22	14.9208	0.0312	47	56.7744	0.0000
23	15.6179	0.0000	48	57.3970	0.0000
24	16.5354	0.3159	49	58.2532	0.0774
25	17.1519	0.4404	50	60.9913	0.4737

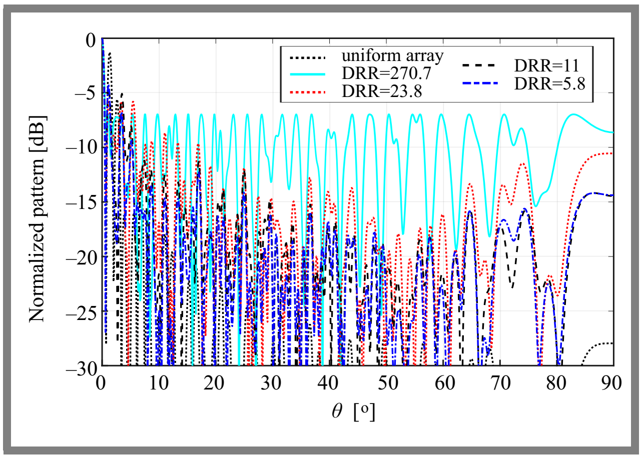

Fig. 3. Patterns of the synthesized arrays with D_0 = 30λ.

Tab. 3. List of element positions and normalized excitations for $N = 25$ element array with $\tau_d = 10$.

n	Position (λ)	w_n	n	Position (λ)	w_n
1	0	0.2012	26	36.4994	1.0000
2	0.8820	0.1335	27	36.9994	0.7551
3	1.4606	0.1050	28	37.5001	0.5570
4	2.0104	0.0899	29	39.2464	0.2014
5	2.5173	0.0856	30	39.8575	0.1748
6	3.4254	0.0908	31	40.3841	0.1801
7	3.9456	0.0970	32	41.4258	0.2285
8	4.4538	0.1055	33	42.1098	0.2608
9	5.1479	0.1204	34	44.2116	0.2098
10	5.8642	0.1315	35	45.0513	0.1605
11	6.9215	0.1346	36	45.6693	0.1337
12	7.4267	0.1365	37	46.2975	0.1179
13	8.3457	0.1447	38	46.8566	0.1134
14	9.3621	0.1563	39	47.4929	0.1144
15	10.3648	0.1478	40	48.0668	0.1159
16	10.9275	0.1238	41	48.8481	0.1191
17	11.4564	0.0926	42	50.5966	0.1194
18	12.0924	0.0540	43	51.3460	0.1136
19	12.8909	0.0174	44	52.6031	0.1268
20	13.5487	0.0110	45	53.5261	0.1574
21	14.4040	0.0640	46	54.2029	0.1809
22	14.9987	0.1582	47	55.5386	0.1786
23	15.4989	0.2835	48	56.0452	0.1657
24	15.9991	0.4580	49	57.9896	0.2040
25	16.4993	0.6843	50	60.5225	0.9048

a list of element positions and the corresponding normalized weights.

Next the optimization problem in (9) is considered under all the constraints. The value of τ_d is set to 10 experimentaly. After optimizing the pattern using the proposed hybrid IWO and convex optimization algorithm, the optimized pattern has a PSLL of –9.10 dB and DRR = 91. The PSLL increased by 0.28 dB (3%) and the DRR decreased by 183 (66.79%) compared to the unconstrained $\|\boldsymbol{w}\|$. The trade-off is obvious between the PSLL and the DRR and will be more obvious as we decrease the value of τ_d. A list of the element position the their normalized excitation is given in Tab. 3.

Next the algorithm is run with $\tau_d = 9$. The obtained PSLL of the normalized pattern is –8.07 dB with DRR of 11.1. This corresponds to an increase in the PSLL of 1.31 (14%) and a decrease in the DRR by –262.9 (96%) compared to the case of unconstrained $\|\boldsymbol{w}\|$. Table 4 lists element positions and the corresponding normalized weights.

Tab. 4. List of element positions and normalized excitations for $N = 25$ element array with $\tau_d = 9$.

n	Position (λ)	w_n	n	Position (λ)	w_n
1	0	0.7657	26	41.0850	0.7198
2	0.5136	0.6078	27	41.5991	0.5804
3	1.1969	0.4334	28	42.3679	0.4144
4	2.5289	0.2078	29	43.1618	0.2939
5	3.7192	0.1142	30	44.2219	0.2020
6	4.2407	0.0971	31	44.7751	0.1786
7	4.9250	0.0902	32	45.5175	0.1657
8	7.0583	0.1243	33	46.6976	0.1701
9	7.8754	0.1425	34	47.5546	0.1794
10	8.5126	0.1553	35	48.1616	0.1848
11	9.3579	0.1694	36	48.7338	0.1875
12	9.9082	0.1769	37	49.6791	0.1863
13	11.3564	0.1896	38	50.3039	0.1817
14	12.3723	0.1920	39	50.9258	0.1748
15	13.3857	0.1890	40	51.5208	0.1666
16	14.7962	0.1812	41	52.3721	0.1530
17	15.3058	0.1807	42	53.0022	0.1423
18	16.1448	0.1893	43	53.6061	0.1319
19	16.7389	0.2072	44	54.4423	0.1184
20	17.5210	0.2531	45	55.0639	0.1104
21	18.3334	0.3374	46	55.6404	0.1060
22	19.3443	0.5083	47	56.5578	0.1101
23	20.0494	0.6770	48	57.3778	0.1328
24	20.5683	0.8282	49	58.2990	0.1918
25	21.0819	1.0000	50	58.9904	0.2677

Finally the algorithm is run for $\tau_d = 8$. The obtained normalized pattern has a PSLL of –6.9 dB and DRR of 5.6. This corresponds to an increase in the PSLL by 2.48 (26.4%) and a decrease in DRR by 268.41 (98%) compared to the case of unconstrained $\|\boldsymbol{w}\|$. A list of the element positions and their normalized excitations are given in Tab. 5. Again, the trade-off is clear between the obtained PSLL and the resultant DRR. It is also obvious that as the $\|\boldsymbol{w}\|$ is constrained to has a lower value, the value of the resultant DRR improves (decreased). The patterns of the three cases of τ_d (i.e. $\|\boldsymbol{w}\|$) are shown in Fig. 2.

4.3. Effect of Distance Between Subarrays

In this section, the distance between the two subarrays is increased to 30λ. It is expected that as the distance between the subarrays increases, the grating lobe level will increase and the FNBW will decrease. For the uniform array with $D_0 = 30\lambda$, the PSLL is –1.27 dB compared to –2.17 dB for

Tab. 5. List of element positions and normalized excitations for $N = 25$ element array with $\tau_d = 8$.

n	Position (λ)	w_n	n	Position (λ)	w_n
1	0	0.4989	26	37.9738	1.0000
2	0.5024	0.4563	27	38.5722	0.9375
3	1.1273	0.4066	28	39.2448	0.8669
4	1.6606	0.3674	29	39.8553	0.8031
5	2.3929	0.3187	30	40.5724	0.7291
6	3.0146	0.2825	31	41.2286	0.6630
7	3.7743	0.2451	32	42.5078	0.5408
8	4.3987	0.2203	33	43.1215	0.4864
9	6.0796	0.1819	34	43.8583	0.4255
10	6.8959	0.1786	35	44.5376	0.3742
11	7.4659	0.1823	36	45.2974	0.3229
12	7.9836	0.1900	37	46.1382	0.2744
13	8.5192	0.2022	38	46.9678	0.2358
14	9.0566	0.2187	39	48.6926	0.1870
15	9.7858	0.2478	40	49.3271	0.1803
16	10.3167	0.2736	41	50.0797	0.1802
17	11.0325	0.3143	42	51.9051	0.2158
18	11.7437	0.3610	43	52.9070	0.2559
19	12.8468	0.4448	44	54.1803	0.3264
20	13.3873	0.4903	45	55.0326	0.3847
21	14.4320	0.5851	46	55.8927	0.4514
22	15.6409	0.7038	47	56.5112	0.5037
23	16.2322	0.7643	48	57.6209	0.6052
24	16.8421	0.8277	49	58.4899	0.6899
25	17.8622	0.9347	50	59.1051	0.7519

the array with $D_0 = 20\lambda$, and the FNBW is 1.4° compared to 1.8° for the array with $D_0 = 20\lambda$. The array is optimized using the proposed algorithm by optimizing both the positions and weights of the array elements for different values of τ_d. For unconstrained $\|\boldsymbol{w}\|$, the obtained PSLL is –6.9770 and the DRR is 270.73. For the case with $\tau_d = 10$, the PSLL is –5.8153 and DRR is 23.79. For $\tau_d = 9$, the PSLL is –5.12 and DRR equals 11. Finally, for $\tau_d = 8$, the PSLL equals –4.4039 and the DRR is 5.8. Figure 3 shows the pattern of the uniform array alongside the patterns for the different obtained DRRs. Table 6 summarizes the obtained results. From Tab. 6, it can be seen that as the distance between the sub-arrays increases, the performance of the array deteriorates.

5. Conclusion

An algorithm based on IWO and convex optimization was presented. The algorithm optimizes the elements' positions and excitations in distributed arrays with two subarrays. Numerical results showed a clear trade-off between the obtained PSLL and the value of DRR. Low DRR resulted in higher PSLL and vice versa.

Tab. 6. PSLL of optimized arrays with different distances between subarrays.

Distance		$D_0 = 20\,\lambda$	$D_0 = 30\,\lambda$
Uniform		–2.17 dB	–1.27 dB
$\tau = \infty$	PSLL	–9.38 dB	–6.977 dB
	DRR	274	270.7
$\tau_d = 10$	PSLL	–9.1 dB	–5.815 dB
	DRR	91	23.8
$\tau_d = 9$	PSLL	–8.07 dB	–5.123 dB
	DRR	11.1	11
$\tau_d = 8$	PSLL	–6.9 dB	–4.404 dB
	DRR	5.6	5.8

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Magdy A. Abdelhay, Ph.D., Assistant Professor
Department of Information and Communication Engineering
https://orcid.org/0000-0003-4244-0840
E-mail: abdelhay@ieee.org
Al-Farqadein University, Basrah, Iraq
https://www.fu.edu.iq/en